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Department of Control and Computer Engineering

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# Design of Experiments for Nonlinear System Identification

Applications of set membership identification to Design of  
Experiments (DoE) and fault detection

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- Accurate **modeling of dynamic systems** is a fundamental step in many fields for simulation, prediction, decision making, fault detection, control design etc.
- **Building an accurate model using the physical laws** governing the system **may not be possible** in several situations, due to the fact that these laws are not sufficiently well known or they are too complex, requiring a computationally expensive model that may be difficult to analyze or to use for design purposes.
- Data-driven **system identification** can be seen as the science of building mathematical models of dynamic systems, using some prior information and measurement data.



- Typically, the **identification process** consists of the following main steps
  - Design of Experiment (DoE);
  - Selection of a suitable parametrized model structure
  - Identification of the model parameters
  - Evaluation of the model quality through some validation analysis
- The command input signal is the only means that can be used in the DoE phase to influence the information content of a dataset to be used for identification (identification dataset or training dataset)
- Regardless of the chosen model structure and identification method, the quality of the DoE determines the accuracy that can be achieved by any identification method.

#### Aim

- Design an experiment giving the maximum information about the system to be identified



The classical approach to fault detection

- Identify a model of the system and to design a filter/observer.
- Use the designed filter/observer to generate online a residual signal.
- The fault is detected when residual exceeds a given threshold.

Drawbacks

- Hard problem in the presence of nonlinear and/or uncertain dynamics
- The choice of threshold may be critical
- Effects of modeling error on the estimation error of the filter designed from approximated model is an open problem.

Set Membership Fault Detection

The approach is based on the direct identification from experimental data of a suitable filter and related uncertainty bounds. These bounds are used to detect when a change (e.g., a fault) has occurred in the dynamics of the system of interest.





- A novel DoE algorithm for input constrained MISO nonlinear dynamic systems based on the set membership identification.
- An innovative approach to fault detection for nonlinear dynamic systems, based on the set membership identification method.
- A Quasi-Local nonlinear set membership identification.



1. Nonlinear Set Membership Identification
2. Set Membership Design of Experiments (SM-DoE)
3. From Design of Experiments to Data-Driven Control Design for Lean NOx Trap (LNT) Regeneration
4. Set Membership Fault Detection for Nonlinear Dynamic Systems
5. Discussion and Conclusions



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# Nonlinear Set Membership Identification



## Nonlinear Set Membership Identification features

1. **No assumptions on the parametric form** of the nonlinear system are needed.
2. **No numerical minimization problems**, thus avoiding the issue of local minima and computation time. Since no optimization problems has to be solved in this approach, it is a suitable tool also for **adaptive identification**; making the model more accurate over time by adding new measurements collected online.  
(Adaptive Nonlinear Controllers, Design of Experiments , Fault Detection)
3. Allows to accurately define the uncertainty of the identified model in a deterministic manner. *Radius of Information*



## Nonlinear Set Membership Identification – Global

Consider a nonlinear discrete-time dynamic system in regression form

$$\begin{aligned} y^{t+1} &= f_o(w^t) \\ w^t &= [y^t \dots y^{t-n_y+1} u^t \dots u^{t-n_u+1}] \end{aligned} \quad (1)$$

Suppose that the function  $f_o$  is unknown but a set of noise corrupted data called *measurement dataset* generated by the system (1) is available

$$\tilde{y}^{t+1} = f_o(\tilde{w}^t) + d^t, t = 1, \dots, T \quad \mathcal{D} \doteq \{\tilde{y}^{t+1}, \tilde{w}^t\}_{t=1}^{T-1}$$

- *Assumption 1* : The noise  $d^t$  is unknown but bounded.  $|d^t| \leq \mu, t = 1, 2, \dots, T.$
- *Assumption 2* : The function  $f_o$  is Lipchitz continues on  $\mathcal{W}$ .  $f_o \in \mathcal{F}(\Gamma)$   
for some  $\Gamma < \infty$ , where

$$\mathcal{F}(\Gamma) \doteq \{f : |f(w) - f(\hat{w})| \leq \Gamma \|w - \hat{w}\|_2, \forall w, \hat{w} \in \mathcal{W}\}.$$



## Optimal Bounds

$$\underline{f}(w) \leq f_o(w) \leq \overline{f}(w), \forall w \in \mathcal{W}$$

$$\overline{f}(w) = \sup_{f \in FSS^T} f(w)$$

$$\underline{f}(w) = \inf_{f \in FSS^T} f(w).$$

$$\overline{f}(w) \doteq \min_{t=1,\dots,T} (\tilde{y}^{t+1} + \varepsilon + \Gamma \|w - \tilde{w}^t\|)$$

$$\underline{f}(w) \doteq \max_{t=1,\dots,T} (\tilde{y}^{t+1} - \varepsilon - \Gamma \|w - \tilde{w}^t\|)$$

## Nonlinear Set Membership Identification – Local

Identify a model  $f'(w)$  using any desired method, Then

$$f_{\Delta}(w) \doteq f_o(w) - f'(w)$$

$$\Delta y^{t+1} = \tilde{y}^{t+1} - f'(\tilde{w}^t), t = 1, \dots, T$$

$$f'(w) + \underline{f}_{\Delta}(w) \leq f_o(w) \leq f'(w) + \overline{f}_{\Delta}(w), \forall w \in \mathcal{W}$$



## Proposed Quasi-Local Approach

Instead of a global constant bound  $\Gamma$  on the gradient of the function, a quasi-local bound is assumed.

Quasi-Local Lipschitz parameters

$$\gamma(w) = \sup_{\hat{w} \in \mathcal{W}, \hat{w} \neq w} \frac{|f_o(w) - f_o(\hat{w})|}{\|w - \hat{w}\|}$$

Global Lipschitz constant

$$\Gamma = \sup_{w \in \mathcal{W}} \gamma(w).$$

Optimal bounds

$$\bar{f}(w) \doteq \min_{t=1, \dots, T} (\tilde{y}^{t+1} + \varepsilon + \gamma(\tilde{w}^t) \|w - \tilde{w}^t\|)$$

Theorems and proofs [\[3\]](#)

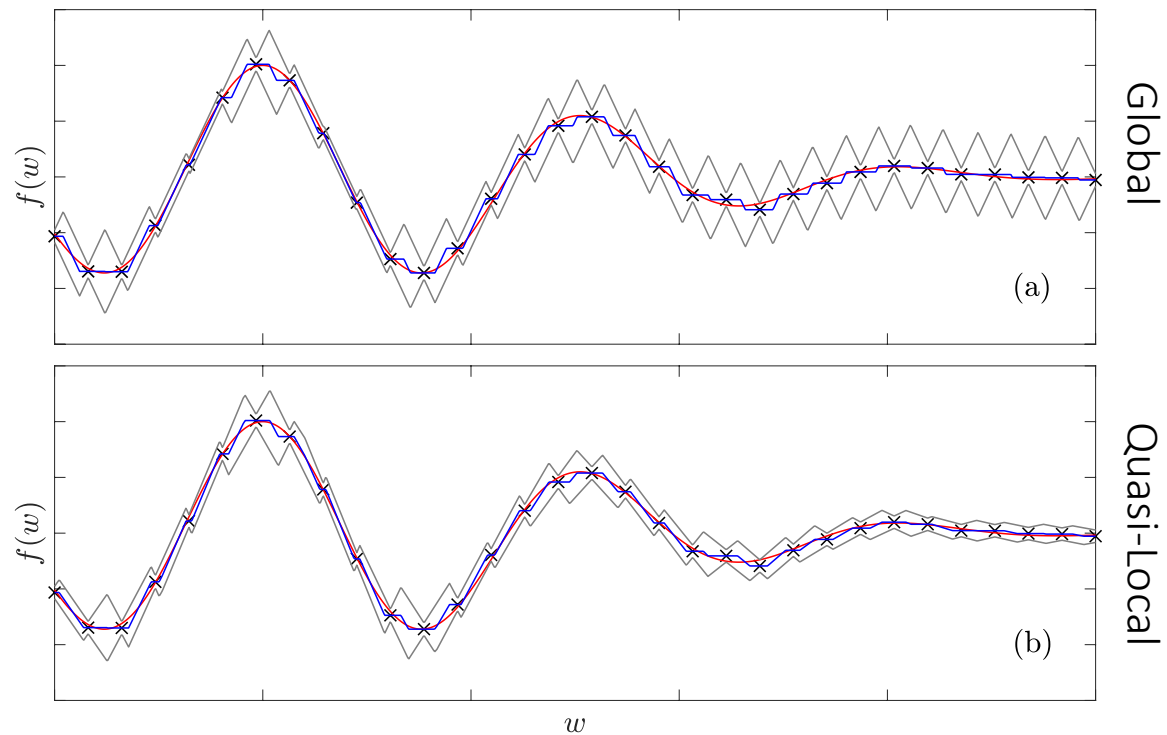
$$\underline{f}(w) \doteq \max_{t=1, \dots, T} (\tilde{y}^{t+1} - \varepsilon - \gamma(\tilde{w}^t) \|w - \tilde{w}^t\|)$$

Advantages

- Less conservative uncertainty bounds with respect to the global approach.
- Unlike the local approach, does not require a preliminary estimate of the function.



## Nonlinear Set Membership Identification







- Error function  $f_e$
- Radius of information  $\mathcal{R}_f^p$  – worst case model error  
An indication of the model uncertainty

$$f_e(w, \mathcal{D}) \equiv f_e(w) \doteq \frac{1}{2}[\bar{f}(w) - \underline{f}(w)]$$

$$\mathcal{R}_{\mathcal{I}}^p = \|f_e(\cdot, \mathcal{D})\|_p$$



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## Set Membership Design of Experiments



## Set Membership Design of Experiments

### Aim

Synthesize an input sequence to apply to the plant to maximize the information extracted from the collected data and thus minimizing the uncertainty of the estimated model.



## Set Membership Design of Experiments

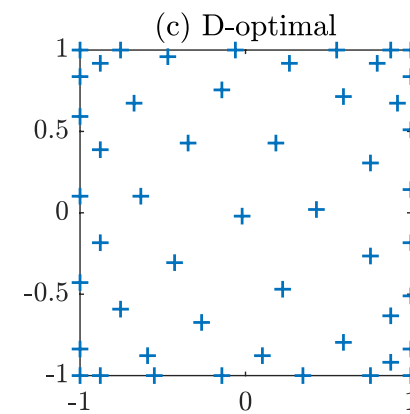
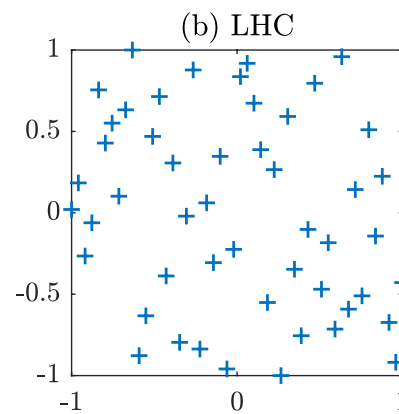
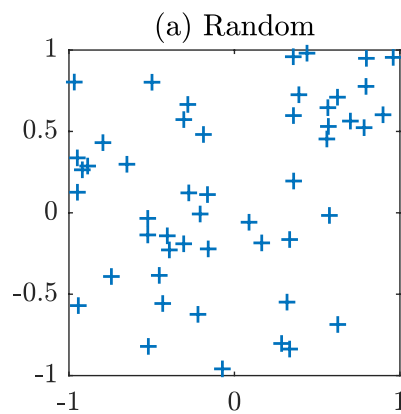
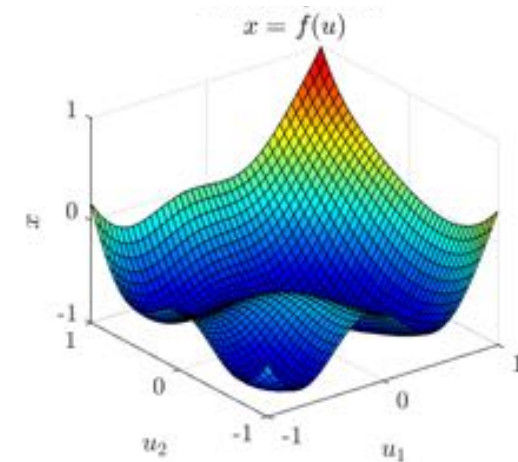
- State of the art

### Static Systems

- Model-free DoE (Space-filling designs)
- Model-based DoE

Random, LHC

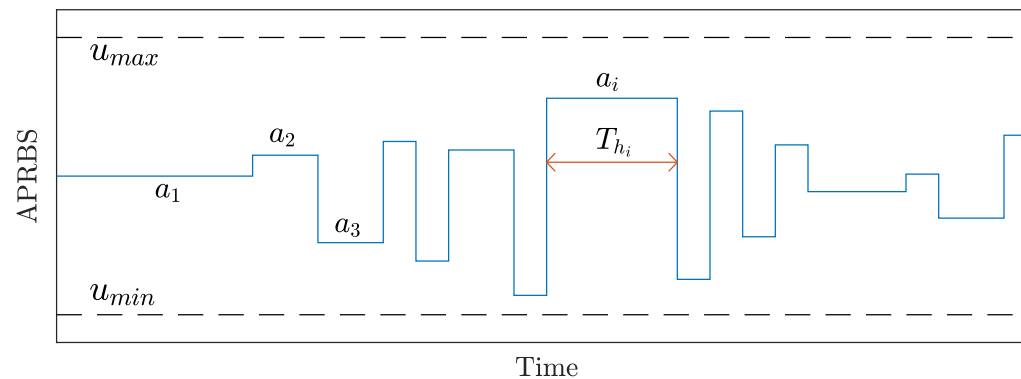
D-Optimal





## Dynamic Systems

- Parameterize a pre-defined excitation signal, and then optimize the signal parameters called the design points, according to different criteria (Random, LHC, D-Optimal, ... ).
- For example, a widely applied excitation signal in industrial identification tasks is the amplitude modulated pseudo random binary signal (APRBS).





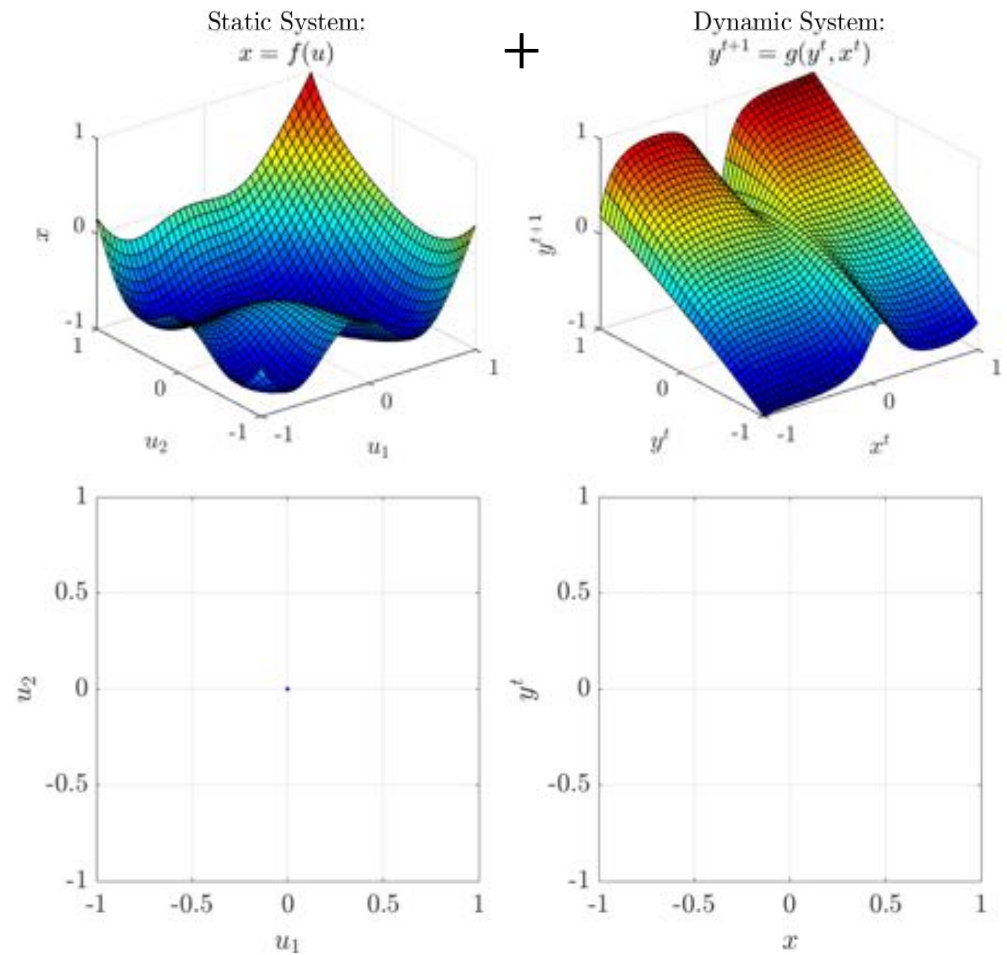
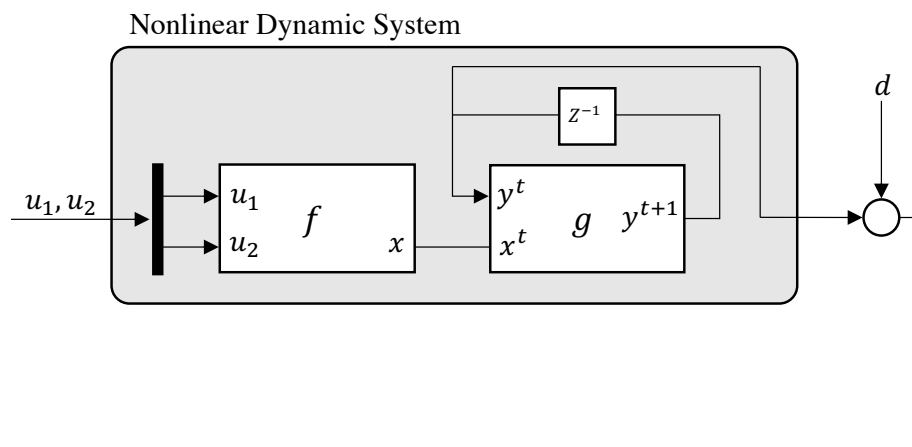
## Set Membership Design of Experiments

- State of the art

### Dynamic Systems

Experiment Duration: 560 seconds

Sampling time: 0.5 seconds





- Simple and adequate to capture steady state behavior.

#### Drawbacks

- Provide no information about the sequence of design points.
- They don't take into account the dynamics of the system.
- Capturing the nonlinear dynamic behavior of the system in the whole regressor domain is a heuristic/arbitrary process.
- No guarantee on the accuracy of the model derived from these methods.

#### Challenge

The exploration of the regressor domain.



## Set Membership Design of Experiments

Knowing which are the regions of the regressor space where the model is most uncertain is a key element to design a proper DoE algorithm.

The DoE algorithm has to be able to generate an input sequence such that the system moves toward those uncertain regions of the regressor space in order to take new measurements.

- **Proposed Approach**

In the set membership framework, the model uncertainty is measured by the radius of information, calculated in some selected norm. We propose a novel set membership design of experiments (SM-DoE) approach for nonlinear systems, aimed at **minimizing the radius of information**.

The algorithm uses a novel adaptive set membership predictive controller (SMPC) that is able to move the system toward most uncertain regions of the regressor space.





## Set Membership Design of Experiments

### Problem definition

Design an input sequence  $U_1^T \doteq \{u^t\}_1^{T-1}$  that, applied to the nonlinear system yields a minimal radius of information

$$\mathcal{R}_{\mathcal{I}}^p = \|f_e(\cdot, \mathcal{D})\|_p$$

$$U_1^{*T} = \arg \min_{U_1^T} \|f_e(\cdot, \mathcal{D})\|_p$$

$$\text{subject to} \quad \tilde{y}^{t+1} = f_o(\tilde{w}^t) + d^t, \quad t = 1, \dots, T-1$$

$$\mathcal{D} = \{\tilde{y}^{t+1}, \tilde{w}^t\}_{t=1}^{T-1}$$

- Challenges

- $f_o$  and  $d^t$  are not known
- The DoE algorithm has to be sequential; at each time step, on the basis of the current and past measured data, the algorithm individuates what is the next point of the regressor domain that the system has to visit.
- It may not be possible to take measurements at a desired point  $w$ , since the function  $f_o$  is a dynamic system and the regressor  $w$  depends on the state of the system.



Consider a static nonlinear system of the form

$$z^t = f_o(w^t), w^t = u^t \in \mathcal{W}.$$

System trajectory depends only on the current input and not on the past input output values.

It is possible to obtain a measurement of the function  $f_o$  at any desired point of the regressor domain  $\mathcal{W}$

### Theorem 1:

Let  $T$  be the number of steps in Algorithm 3 and  $\mathcal{R}_j(t)$  be the radius of information computed at time  $t$ . Then, there exists a  $T$  such that  $\mathcal{R}_j(t) \leq \mu$ ,  $\forall t \geq T$ .

Proof: [3]

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### Algorithm 3 Static Set-Membership DoE

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(1) Choose the initial regressor  $w^1$  (e.g., the center of the regressor domain  $\mathcal{W}$ );  
Measure  $\tilde{z}^1 = f_o(w^1)$ ;  
Define the measurement dataset  $\mathcal{D} = \{\tilde{z}^1, w^1\}$ .

(2) While  $t < T$ , solve the optimization problem

$$\begin{aligned} w_M^t &= \arg \max_{w \in \mathcal{W}} f_e^t(w, \mathcal{D}); \\ w^t &\in w_M^t \end{aligned} \quad (25)$$

Measure  $\tilde{z}^t = f_o(w^t)$ ;  
Add  $\tilde{z}^t$  and  $w^t$  to the dataset  $\mathcal{D} := \mathcal{D} \cup \{\tilde{z}^t, w^t\}$ ;  
 $t := t + 1$ .

The vector  $w^t$  is any point in  $w_M^t$  and  $f_e^t$  is the error function (18) computed at time instant  $t$ .

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Now suppose that we have a dynamic system. unlike the static case, it is not possible to evaluate the function at any desired point  $w$ , Since the regressor depends not only on the current input but also on past input and output values.

$$\begin{aligned} y^{t+1} &= f_o(w^t) \\ w^t &= [y^t \dots y^{t-n_y+1} u^t \dots u^{t-n_u+1}] \end{aligned} \quad (1)$$

The idea here is to use an algorithm similar to Static SM-DoE to generate desired reference points  $w^r$  in combination with an MPC controller making the system visit the desired point  $w^r$ .

The MPC approach that we propose is novel and is called set membership model predictive control (SMPC)

Other set membership predictive control laws assume that sufficient set of data is available.



State space representation of (1)

$$\begin{aligned} x^t &= [y^t \dots y^{t-n_y+1} \quad u^{t-1} \dots u^{t-n_u+1}] \\ &= [x_{(1)}^t \dots x_{(n_y)}^t \quad x_{(n_y+1)}^t \dots x_{(n_y+n_u)}^t] \\ w^t &= [x_{(1)}^t \dots x_{(n_y)}^t \quad u^t \quad x_{(n_y+1)}^t \dots x_{(n_y+n_u)}^t] \end{aligned}$$

$$x^{t+1} = f_o(x^t, u^t)$$

$$f_o(x^t, u^t) = [f_o(w^t) \quad x_{(1)}^t \dots x_{(n_y-1)}^t \quad u^t \dots x_{(n_y+n_u-1)}^t]$$

$$x^{t+1} = f_o(x^t, u^t) = f_c(x^t, u^t) + [e^t \quad 0 \dots 0]$$

$$|e^t| \leq f_e(w^t) \leq \mathcal{R}_I^\infty \quad \forall w \in \mathcal{W} \quad (2)$$



The set of all possible state values at time  $t + k$  that originate from the generic “initial” state  $x^t$  at time  $t$  by applying the input sequence  $U_t^k$  to the system, is defined as:

$$\begin{aligned}\mathcal{S}(x^t, U_t^k) &= \{ \hat{x}^{t+k} : \\ &\hat{x}^{t+n+1} = f_c(\hat{x}^{t+n}, u^{t+n}) + [e^{t+n} \quad 0 \quad \dots \quad 0], \\ &|e^{t+n}| \leq f_e(\hat{w}^{t+n}), \forall n \in [0, k-1] \}.\end{aligned}$$

The size of the set  $\mathcal{S}$  is an indication of the uncertainty of a trajectory

The state of the system at time  $t + k$  obtained starting from a generic “initial” state  $x^t$  and applying the input sequence  $U_t^k$  is defined as

$$\begin{aligned}\mathcal{S}_o(x^t, U_t^k) &\doteq x^{t+k} : \\ x^{t+n+1} &= f_o(x^{t+n}, u^{t+n}) \quad \forall n \in [0, k-1].\end{aligned}$$

$$\mathcal{S}_o(x^t, U_t^k) \in \mathcal{S}(x^t, U_t^k)$$



Suppose we want to take a measurement at  $w^r$  or its equivalent  $(x^r, u^r)$ . Where the uncertainty amplitude is  $f_e(w^r)$ . We want to derive the system state  $x^t$  to a neighborhood of  $x^r$ , called reference set  $\mathcal{X}_r$

Reference set 
$$\mathcal{X}_r \doteq \{x : \Gamma \|x - x^r\|_2 + \mu < \lambda f_e(w^r), \lambda \in (0, 1]\}$$

When the state is inside  $\mathcal{X}_r$  i.e.  $x^t \in \mathcal{X}_r$ , by applying  $u^r$  as input to the system and adding the new measurement to the dataset  $\mathcal{D}$ , the uncertainty  $f_e(w^r)$  will be reduced by at least a factor of  $\lambda$ .

*Assumption 3: (controllability)*

$$\forall x^t, x^r \in \mathcal{X}, \exists K < \infty, \exists U_t^k \in \mathcal{U} : \\ \mathcal{S}_o(x^t, U_t^k) = x^r \text{ for } k < K.$$

*Assumption 4: (boundedness)*

$$\forall u^t \in \mathcal{U}, \forall t \geq 0 : \quad x^t \in \mathcal{X}$$



The optimization problem to be solved in the SMPC approach is:

$$\begin{aligned} J^*(x^t, x^r, i) &= \max_{U_t^i} J(x^t, x^r, U_t^i) \\ \text{subject to } U_t^i &\in \mathcal{U} \\ x^r &\in \mathcal{S}(x^t, U_t^i) \end{aligned} \quad (3)$$
$$J(x^t, x^r, U_t^i) = \sum_{n=1}^i \text{diam}(\mathcal{S}(x^t, U_t^n))$$

Where:  $i = \min \{i \in \mathbb{N} : i < K, \exists U_t^i \text{ such that } x^r \in \mathcal{S}(x^t, U_t^i)\}$

The controller is implemented according to receding horizon strategy, the control law indicated as  $u^t = \mathcal{K}(x^t, x^r)$ , means solving (3) and applying the first element of the maximizer  $U_t^{*i}$  as control action  $u^t$  and adding new measurement to the dataset  $\mathcal{D}$ .



### Theorem 2:

Let Assumptions 3 and 4 hold. Starting from any initial state  $x^t \in \mathcal{X}$ , the state of the system controlled by the feedback law  $S_o(x^t, \mathcal{K}_t^k)$ , will visit a point inside reference set  $\mathcal{X}_r$  in finite time. That is,

$$\forall x^t, x^r \in \mathcal{X}, \exists K < \infty : S_o(x^t, \mathcal{K}_t^k) \in \mathcal{X}_r \text{ for some } k < K.$$

Proof: [\[3\]](#)





## Set Membership Design of Experiments

### Corollary:

For any desired radius of information  $\mathcal{R}_d \geq \mu$ ,  
there exist a finite number of steps  $T$  Algorithm 4  
such that  $\mathcal{R}_j(t) \leq \mathcal{R}_d, \forall t \geq T$ .

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### Algorithm 4 Dynamic Set Membership DoE

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- (1) Select a reference regressor  $w^r$  to be visited which has a high uncertainty and its equivalent pseudo-state  $x^r$  is close to the estimated state:

$$w^r, x^r = \arg \min_{w^r \in \mathcal{W}, x^r \in \mathcal{X}} (\|\hat{x}^{t+1} - x^r\|_2 + \frac{\delta}{f_e(w^r)}). \quad (57)$$

- (2) Compute  $\mathcal{X}_r$  according to (39) with a suitable  $\lambda$ .
- (3) Apply the following criterion:

$$\begin{aligned} \text{if} \quad & \hat{x}^{t+1} \in \mathcal{X}_r \\ \text{then} \quad & u^t = u^r \in w^r \\ \text{else} \quad & u^t = \mathcal{K}(x^r, x^t). \end{aligned}$$

- (4) Evaluate  $\tilde{y}^{t+1} = f_o(\tilde{w}^t) = f_o(x^t, u^t)$ .
  - (5) Add  $\tilde{y}^{t+1}$  and  $\tilde{w}^t$  to the dataset  $\mathcal{D} := \mathcal{D} \cup \{\tilde{y}^{t+1}, \tilde{w}^t\}$ .
  - (6) Update  $\gamma$  and  $\Gamma$  according to Algorithm 2.
  - (7) Set  $t := t + 1$  and go to step (1).
-

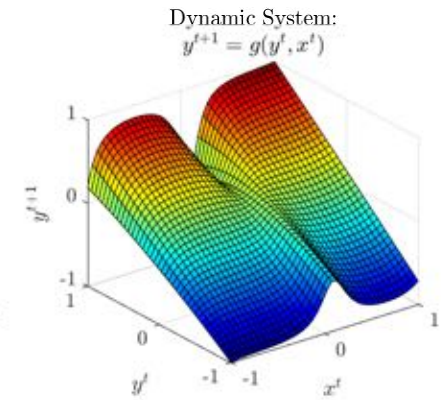
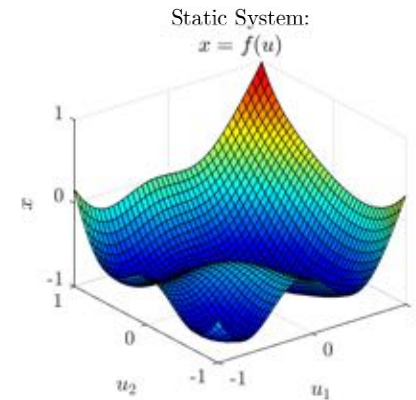
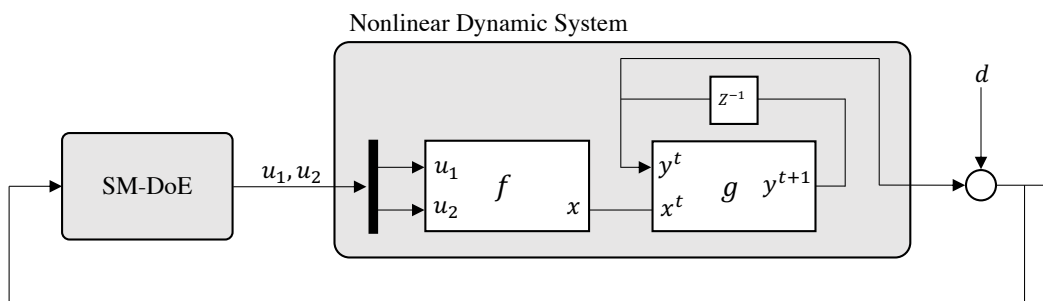


## Set Membership Design of Experiments

Example: Relevant to the behavior of a combustion engine [4]

$$\dot{y} = g(y, x) = 2x / (2.4 \cos(10x + 4) - 0.5y + 3.3).$$

$$x = f(u_1, u_2) = \cos(9\sqrt{u_1^2 + u_2^2} + 2) + 0.5 \cos(11u_1 + 2) + 15((u_1 - 0.4)^2 + (u_2 - 0.4)^2)^2.$$





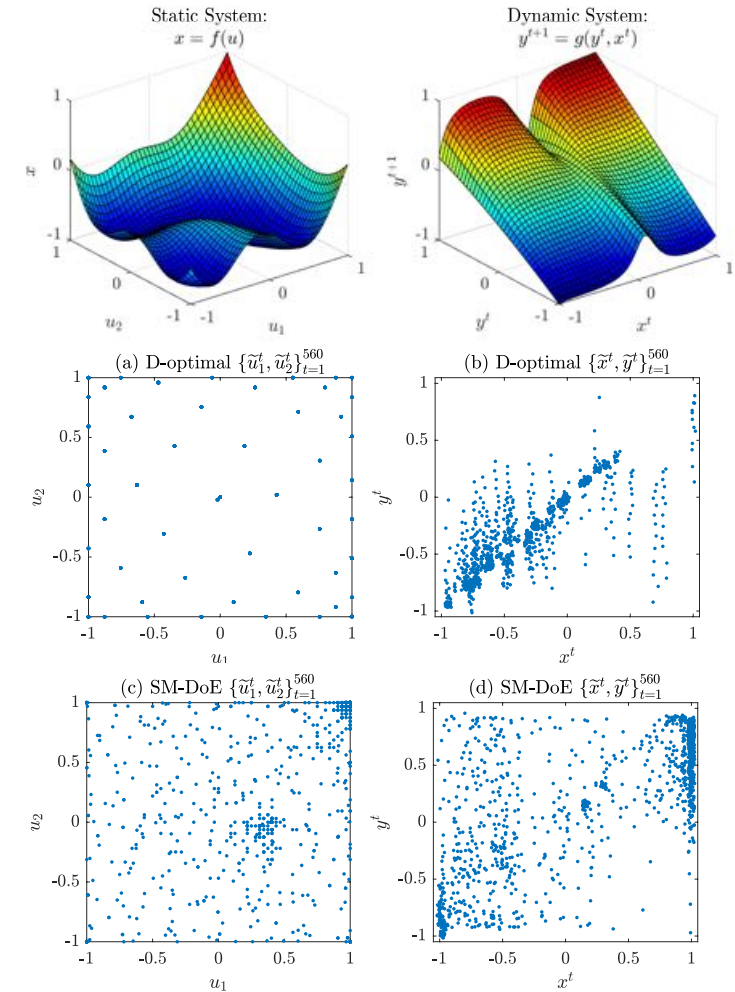
## Set Membership Design of Experiments

Experiment Duration: 560 seconds  
Sampling time: 0.5 seconds

- Three different DoE methods are compared (Random, LHC, D-Optimal)
- For each one, 10 different APRBS signals are constructed with random sequence of the design points.

### Identified Model Accuracy (Neural Network model)

Inputs	FIT	RMSE
Random	$0.69 \pm 0.03$	$0.149 \pm 0.017$
LHC	$0.68 \pm 0.04$	$0.155 \pm 0.022$
D-Optimal	$0.76 \pm 0.01$	$0.115 \pm 0.008$
SM-DoE	0.91	0.043

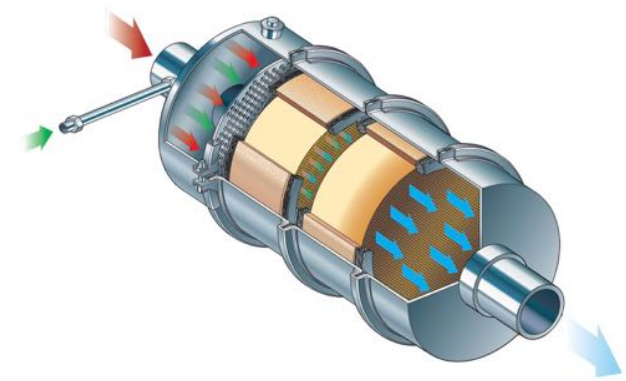




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# From Design of Experiments to Data-Driven Control Design for Lean NOx Trap Regeneration

- Diesel engines:
  - High efficiency
  - Higher emissions compared to SI Engines
- NOx emission reduction:
  - LNT
  - SCR
- Lean NOx Trap (LNT):
  - Lean NOx Trap is an after-treatment technology that is used to reduce the NOx emissions of lean burn diesel engines
  - The basic concept of LNT is to store NOx during lean conditions and release the stored NOx in rich conditions to react with available reductants to reduce NOx emissions.





- Two main operating conditions:
  1. **Storage** or Adsorption phase
    - Engine is in lean condition
    - NOx is Stored Chemically in LNT
  2. **Regeneration** or Purge phase
    - Engine is in rich condition (extra fuel is injected)
    - NOx is released from the catalyst then react with CO, HC and H<sub>2</sub> to make nitrogen.





In order to regenerate the LNT, extra fuel is injected in the cylinder to create a rich condition, and this causes a higher fuel consumption. The goal is to determine the best timing for regeneration in order to reduce the fuel consumption and the NOx emissions.

- Regeneration Control Objective
  1. Minimize Fuel Penalty
  2. Keep NOx emissions under the regulation level
- Problems:
  1. Deriving an accurate model, able to describe the highly nonlinear dynamics of an LNT
  2. Designing an effective LNT control strategy using this model



## Data-driven model predictive control ( $D^2 - MPC$ ) for LNT Regeneration

- Does not require a physical model of the plant
- Based on a prediction model, directly identified from experimental data.
- The regeneration timing is computed through an on-line optimization algorithm, which uses the identified model to predict the the amount of NO<sub>x</sub> stored in the LNT.

First step is to identify a model for the LNT

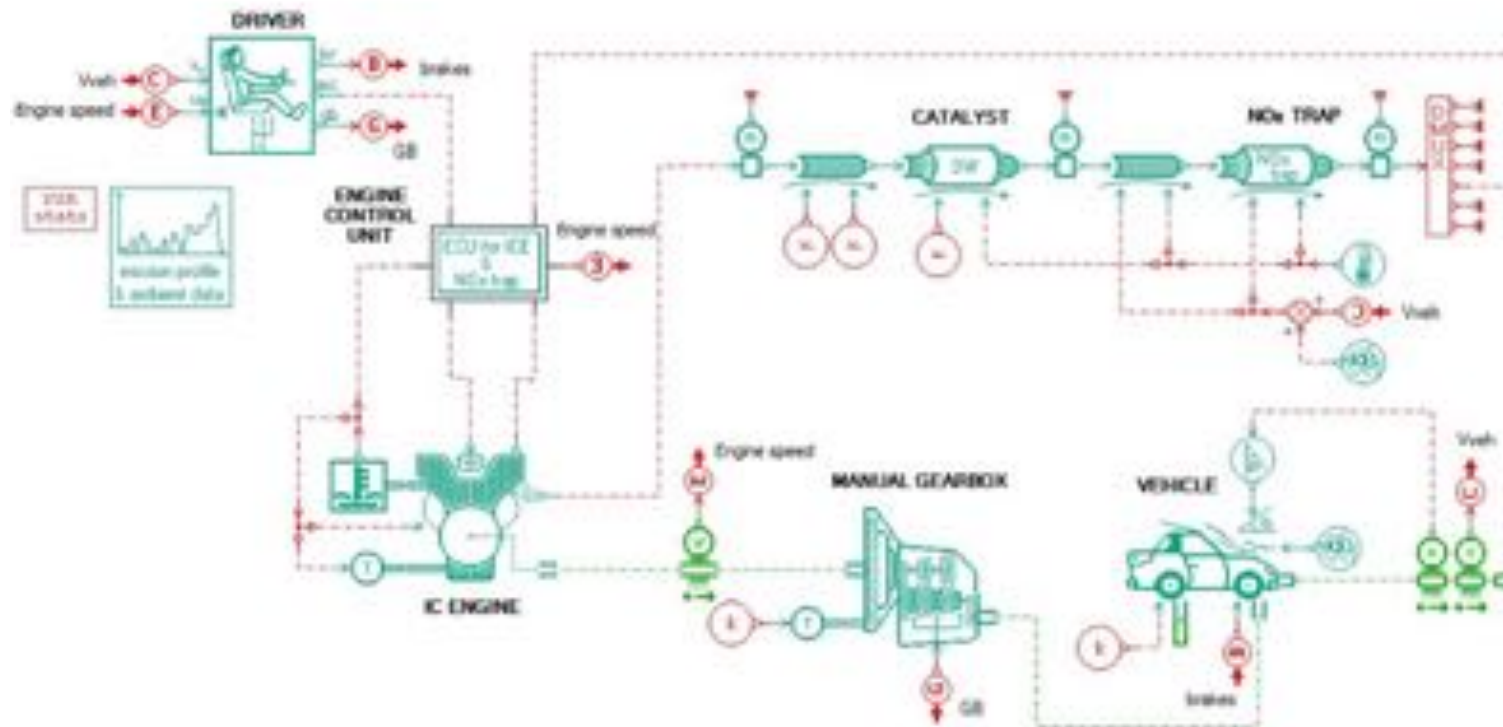
Apply the Set Membership DoE algorithm (SM-DoE) in order to minimize the experimental effort and acquire a rich dataset for identification by capturing the nonlinear behavior of the LNT in the whole working domain of the system.





## Design of Experiments for LNT

- AMEsim model







## Design of Experiments for LNT

$$y^{t+1} = f(y^t, u^t)$$

$$u = [u_{Temp} \quad u_{\phi} \quad u_{\dot{m}_{EG}} \quad u_{\dot{m}_{NO_x}}]$$

$y$  :  $NO_x$  stored quantity

$u_{Temp}$  : LNT wall temperature

$u_{\phi}$  : fuel-air equivalence ratio

$u_{\dot{m}_{EG}}$  : exhaust gas flow rate

$u_{\dot{m}_{NO_x}}$  : exhaust  $NO_x$  flow rate

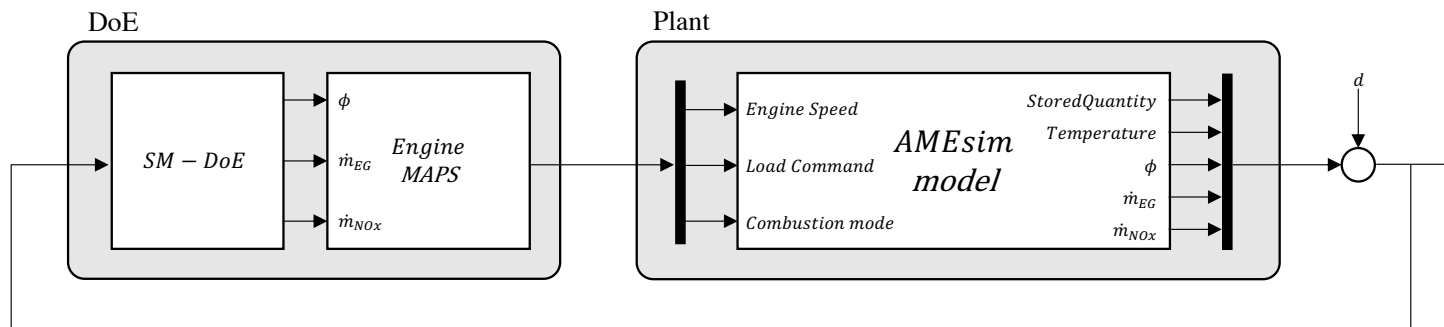
## Engine Maps

$$T_{eng} = f_T(V_{eng}, L_{cmd}, C_{mode})$$

$$\phi = f_{\phi}(V_{eng}, T_{eng}, C_{mode})$$

$$\dot{m}_{EG} = f_{\dot{m}_{EG}}(V_{eng}, T_{eng}, C_{mode})$$

$$\dot{m}_{NO_x} = f_{\dot{m}_{NO_x}}(V_{eng}, T_{eng}, C_{mode})$$





## Design of Experiments for LNT

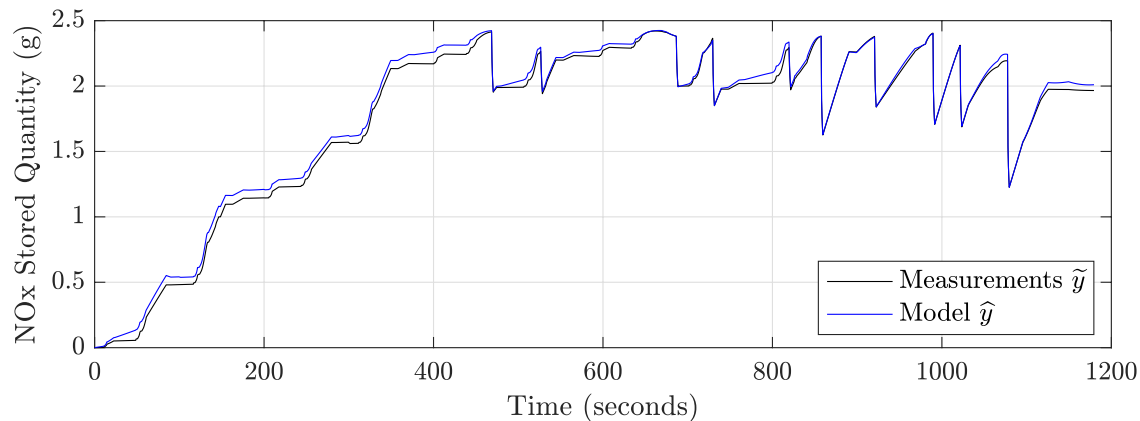
Two Neural network models were identified using the set of data generated by SM-DoE algorithm and data generated by APRBS-Random strategy.

The model works in simulation

$$\hat{y}^{t+1} = f(\hat{y}^t, u_{Temp}^t, u_{\phi}^t, u_{\dot{m}_{EG}}^t, u_{\dot{m}_{NOx}}^t)$$

Validation set: 10 set of data during NEDC driving cycle with random initial LNT temperature and random initial stored quantity.

Inputs	Duration (Samples)	RMSE	FIT	$\mathcal{R}_I^\infty$	$\mathcal{R}_I^2$
APRBS-Random	20000 - 2.8 hours	0.11	0.65	1.03	0.22
SM-DoE	3000 - 25 minutes	0.03	0.90	0.36	0.07





Control Objectives:

- Minimize fuel penalty.
- Maximize amount of NOx removed from the trap in each regeneration.

$$u^t = \begin{cases} 0 & t < t_1, t > t_2 \\ 1 & t_1 \leq t \leq t_2 \end{cases}$$

$$\hat{y}^{t+t_1} - \hat{y}^{t+t_2}$$

$$FP(t_1, t_2) = \sum_{t=t_1}^{t_2} (\phi_{reg}^t - \phi_{eng}^t) \frac{\dot{m}_{EG}^t}{AFR_{stoich}}$$

$$J(t_1, t_2) \equiv J(t, t_1, t_2) = \frac{FP(t_1, t_2)}{\hat{y}^{t+t_1} - \hat{y}^{t+t_2}}.$$

---

#### Algorithm 5 LNT Regeneration Timing Control

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- 1: Compute  $V_{eng}^{t+k}$ ,  $T_{eng}^{t+k}$  for  $k \in [0, K_M]$  according to (4).
- 2: Solve optimization problem:

$$(t_1^*, t_2^*) = \arg \min_{0 \leq t_1 < t_2 \leq k_M} J(t_1, t_2) \quad (58)$$

by computing  $\phi^{t+k}$ ,  $\dot{m}_{EG}^{t+k}$ ,  $\dot{m}_{NOx}^{t+k}$  according to (5) that is a function of  $V_{eng}^{t+k}$ ,  $T_{eng}^{t+k}$ ,  $t_1$  and  $t_2$ . And, by simulating the LNT model starting from  $\hat{y}^t$  to  $\hat{y}^{t+k}$ .

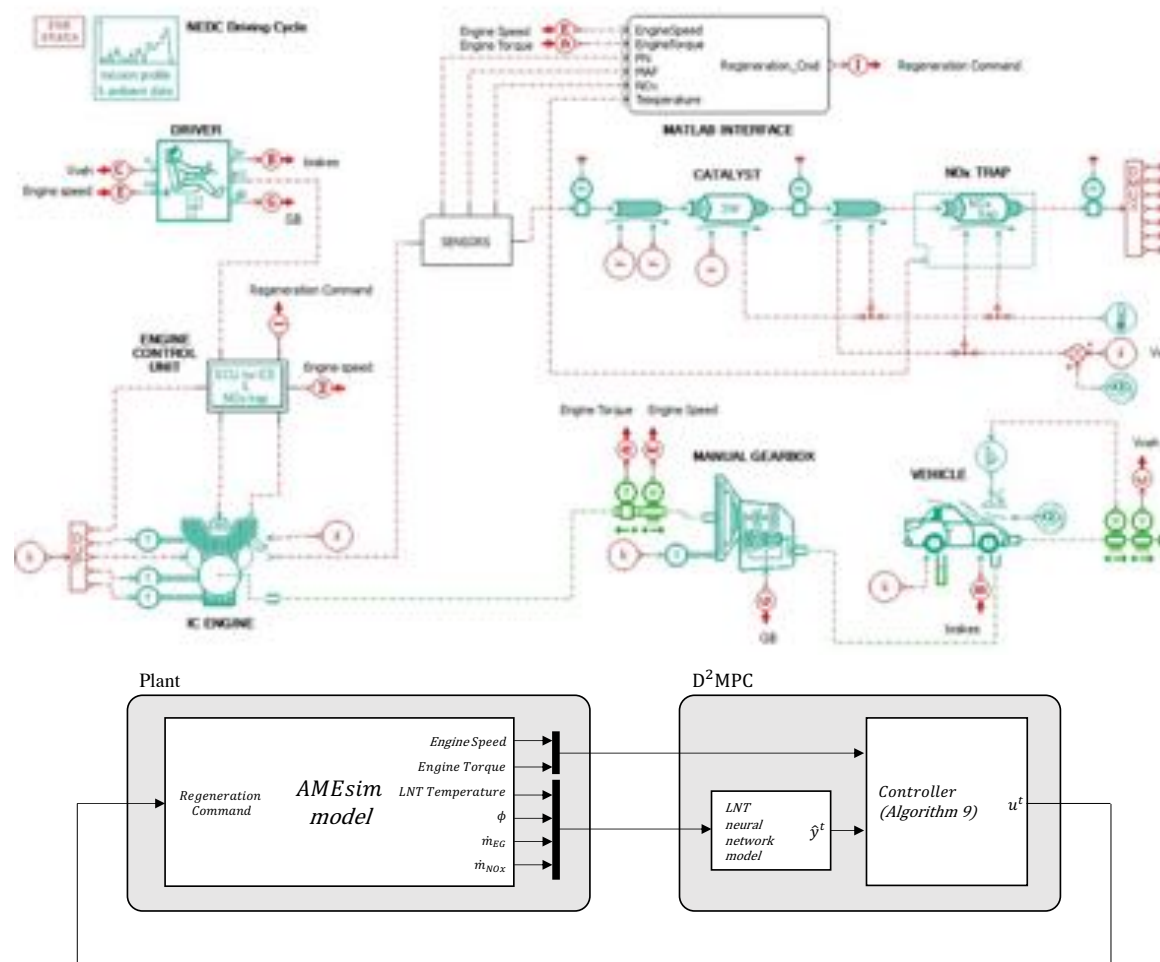
- 3: **if**  $J(t_1^*, t_2^*) \leq \text{Threshold}$  **then**
  - 4:  $u^* = u_t$
  - 5: **else**
  - 6:  $u^* = 0$
  - 7: **end if**
  - 8: Set  $t = t + t_s$  and go to step (1).
-





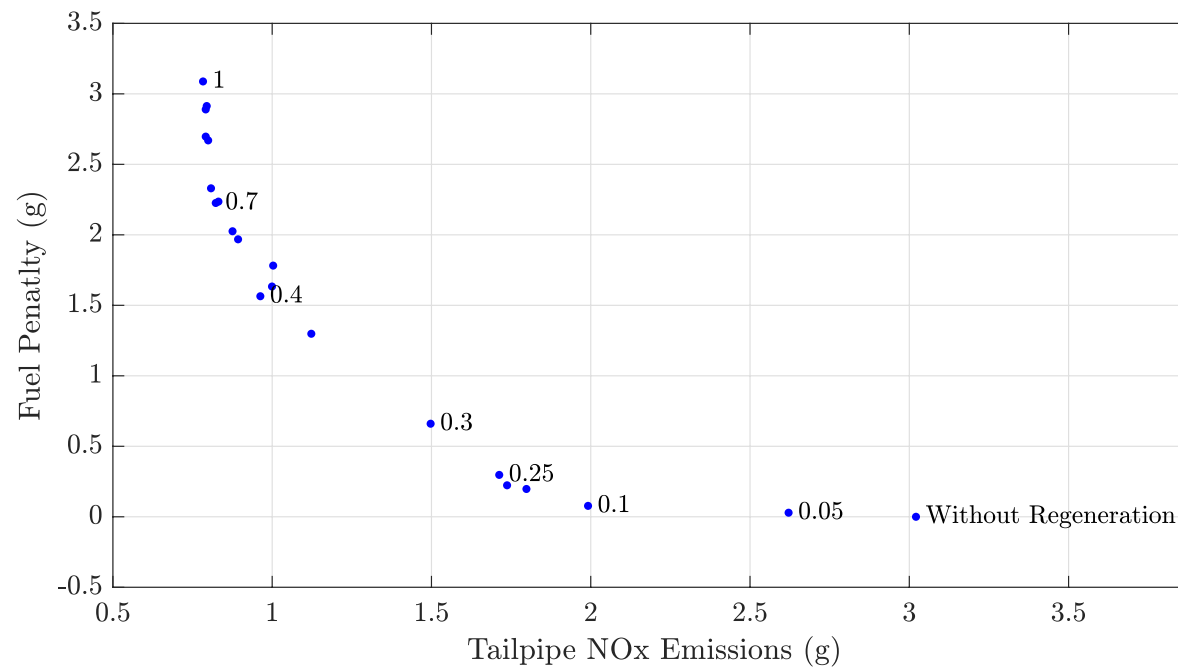
## Design of Experiments for LNT

## - Data-Driven MPC for LNT





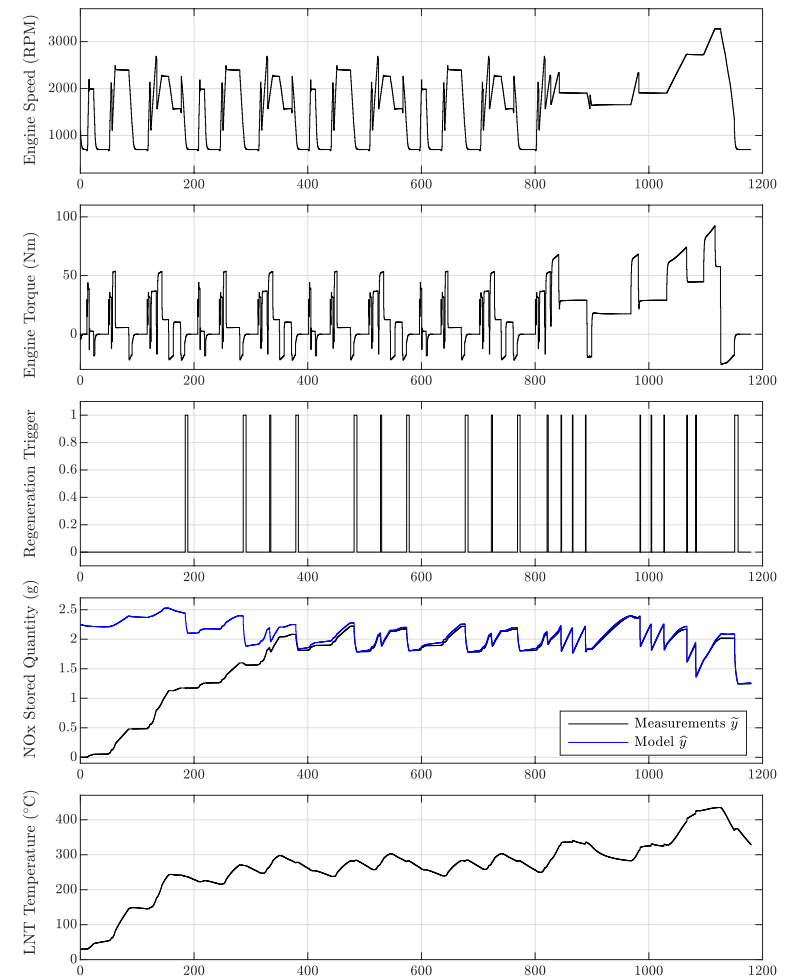
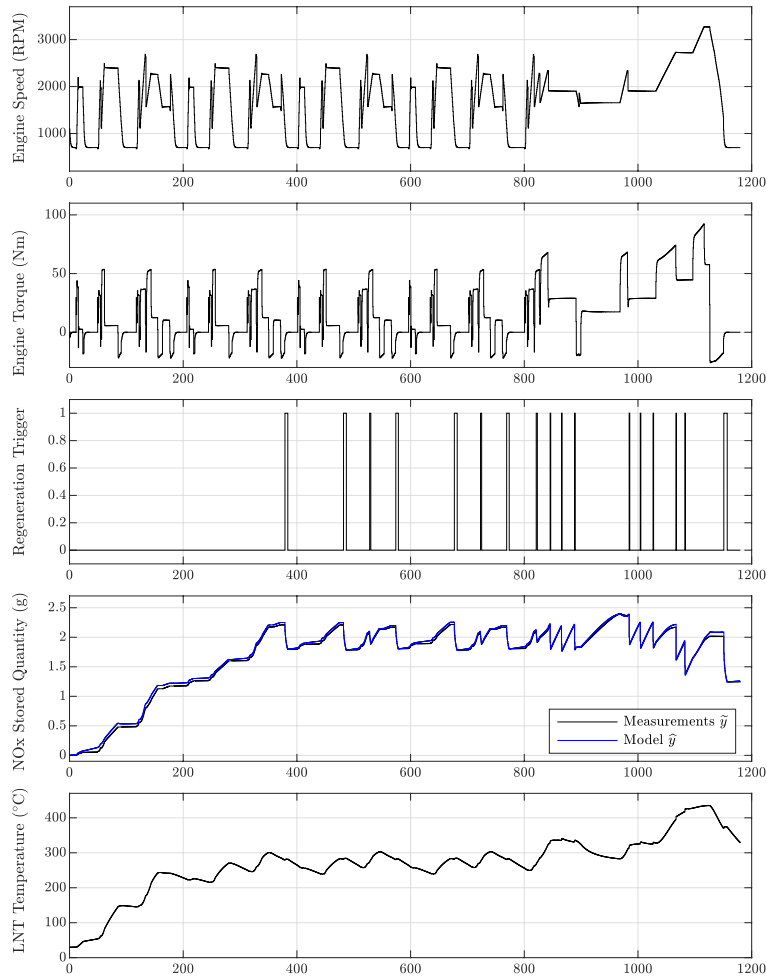
### NEDC Driving Cycle



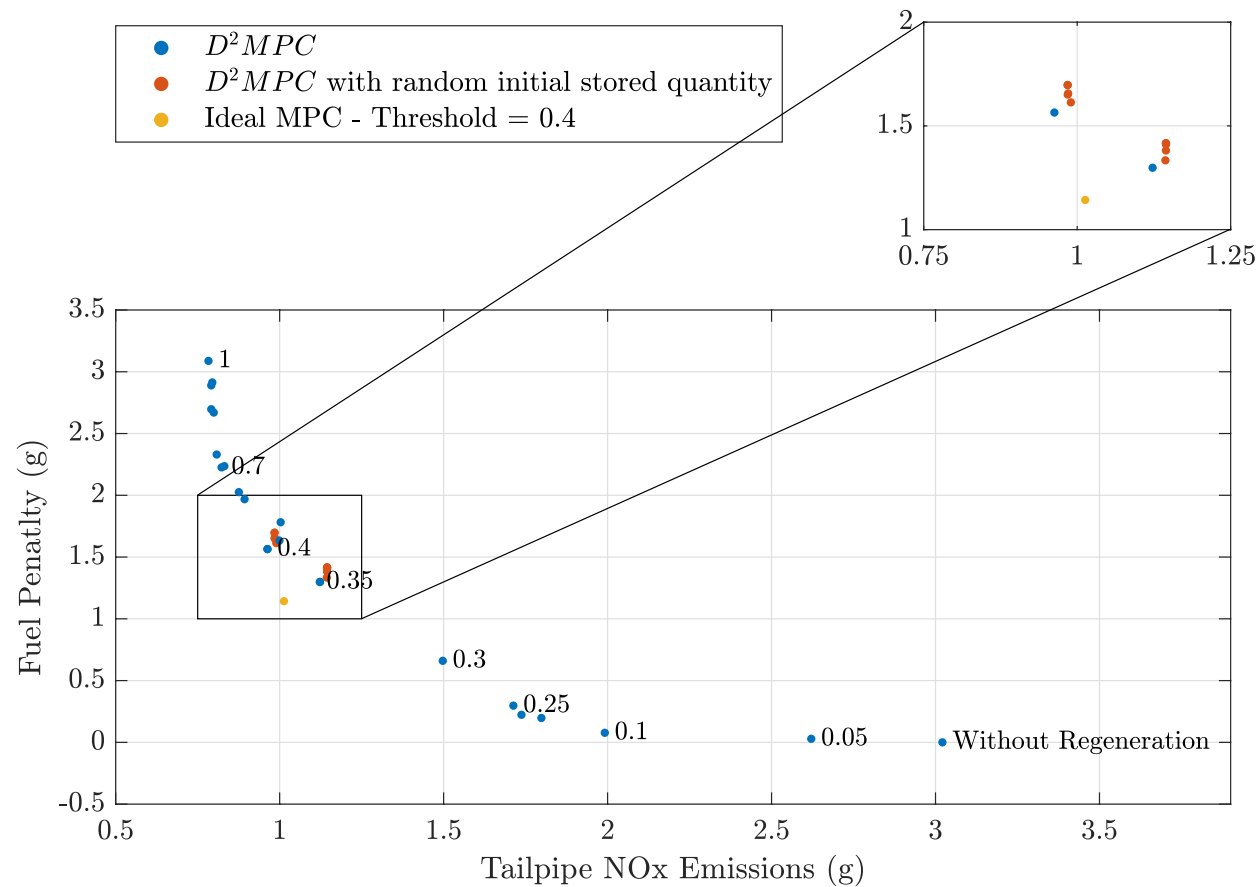


## Design of Experiments for LNT

## - Simulation Results









## Design of Experiments for LNT

### Conclusion

- Avoid complex modeling of the LNT dynamics.
- Very low experimental effort.
- Very efficient regenerations.
- The DoE algorithm could also be used for engine modeling and calibration.



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# Set Membership Fault Detection for nonlinear dynamic systems



## Set Membership Fault Detection

Based on direct identification from experiment data of a suitable filter and related uncertainty bounds.

The bounds are used to detect when a change (e.g., a fault) has occurred in the dynamics of the system.

$$y^t = f_o(x^t) + d^t$$
$$x^t = (y^{t-1}, \dots, y^{t-n_y}, u^{t-1}, \dots, u^{t-n_u})$$

$$\mathcal{D} = \{\tilde{x}^t, \tilde{y}^t\}_{t=1}^T$$

$$\underline{f}(\tilde{x}^t) \leq f_o(\tilde{x}^t) \leq \overline{f}(\tilde{x}^t), \forall t.$$

- Avoids the utilization of complex modeling and filter design procedures since the filter/observer is directly identified from data.
- Does not require to choose any threshold (as typically done in many “classical” techniques).
- Not affected by under modelling problems.
- Can be adaptive (quasi-local approach) – useful in systems where the dynamics changes over time.



## Set Membership Fault Detection

- Offline operations

1. Define the measurement dataset  $\mathcal{D}$ .
2. Estimate the noise bound  $\mu$  according to Algorithm 1.
3. In the case of the local approach, estimate a preliminary approximation  $f_*$ .
4. Estimate the Lipschitz parameters according to Algorithm 2.  $\hat{\Gamma}; \hat{\gamma}(x); \hat{\gamma}_\Delta(x)$ .

- Online operations

1. At each time step:

$$\text{If} \quad \tilde{y}^t > \bar{f}(\tilde{x}^t) + \varepsilon^t \quad \text{or} \quad \tilde{y}^t < \underline{f}(\tilde{x}^t) - \varepsilon^t$$

$$\text{Then} \quad \textit{Fault} = 1$$

$$\text{Else} \quad \textit{Fault} = 0$$

2. In the case of the adaptive algorithm, update the set membership model according to Algorithm 5 if no fault has accrued in the system and the system is not recovering from a fault.



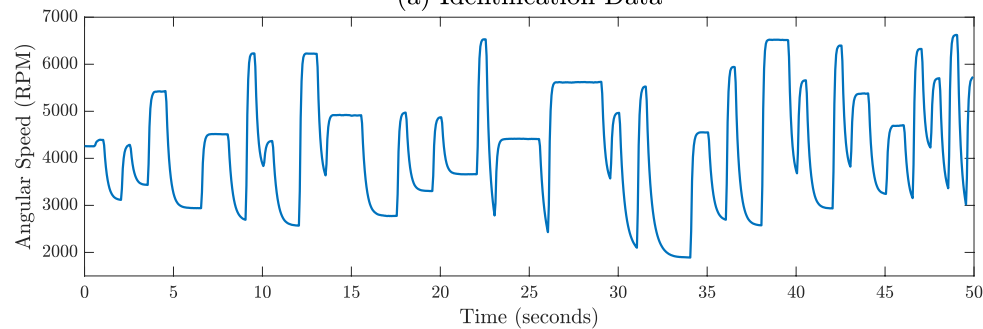
## Set Membership Fault Detection

$$y^t = f_o(x^t) + d^t$$

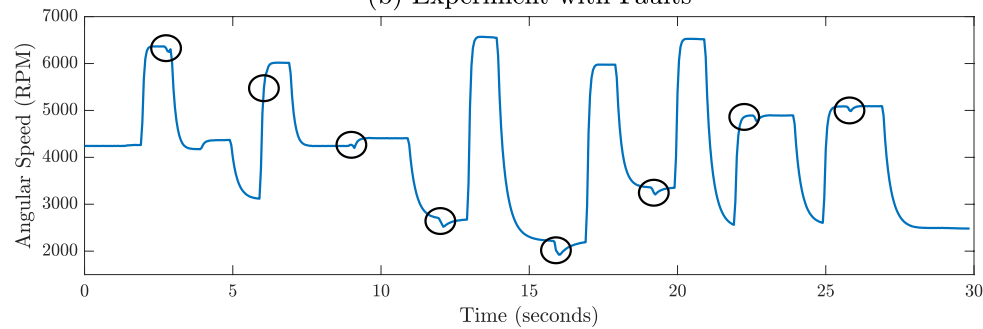
$$\mathcal{D} = \{\tilde{x}^t, \tilde{y}^t\}_{t=4}^{1000}$$

$$\tilde{x}^t = [\tilde{y}^{t-1} \quad \tilde{y}^{t-2} \quad \tilde{u}^{t-2} \quad \tilde{u}^{t-3}]$$

(a) Identification Data

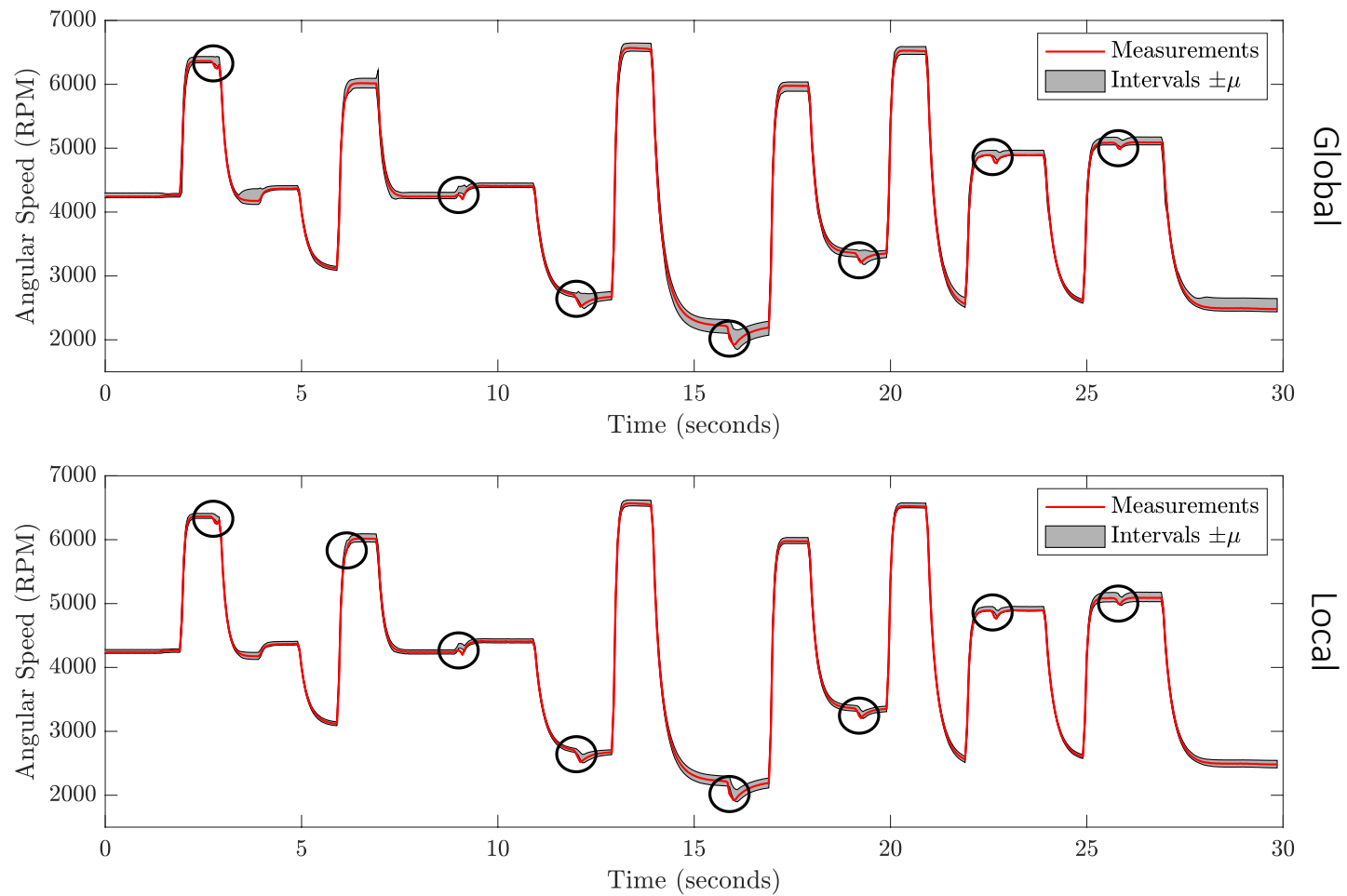


(b) Experiment with Faults



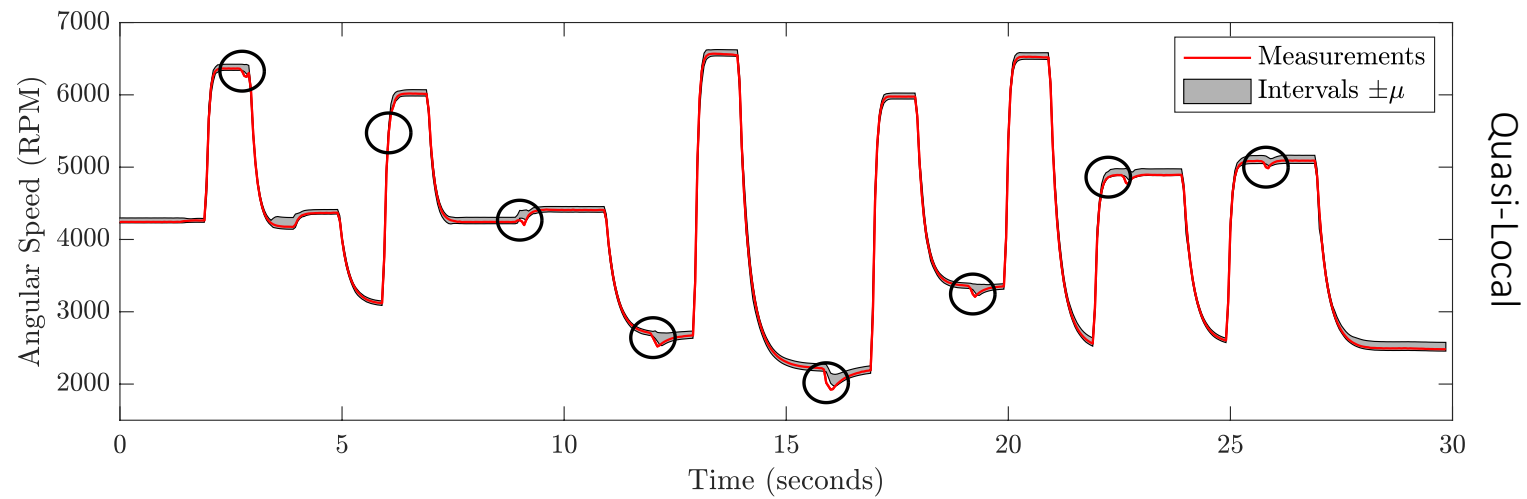


## Set Membership Fault Detection





## Set Membership Fault Detection

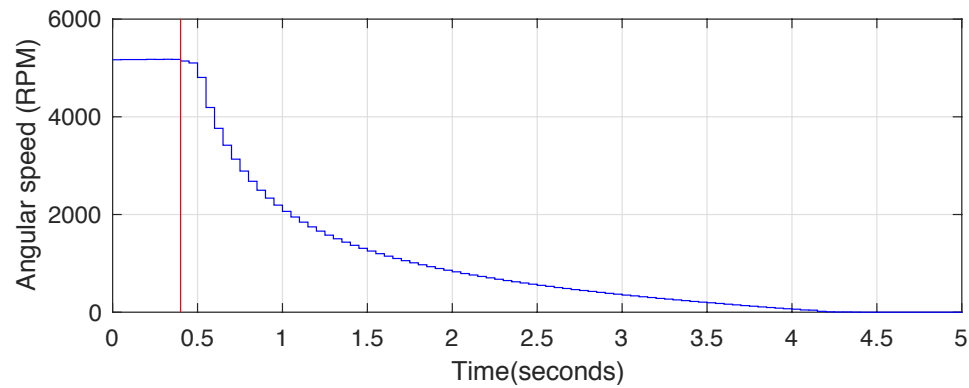






## Set Membership Fault Detection

Turn off the motor as soon as it detects a fault.





## Conclusion

The main goal was to develop a general systematic methodology for design of experiment for nonlinear systems that can be applied to a wide range of applications.

- Quasi-Local Nonlinear Set Membership Identification
  - Less conservative bounds compared to global approach.
  - Doesn't need any preliminary estimate of the function.
  - Can be adaptive (useful for data-driven adaptive controllers, fault detection,...)
- Set Membership Design of Experiments (SM-DoE)
  - A novel DoE algorithm MISO nonlinear systems.
  - A novel SMPC controller to move the system toward most uncertain regions of the domain.
  - Guarantee any desired worst-case model error larger than the measurement error in finite time experiment.
  - Case study in the automotive field to show the effectiveness of the approach and its potential in view of real-world applications where experiments are expensive and/or a very accurate model is desired.



## Conclusion

- Set Membership Fault Detection – Quasi Local approach
  - Fast and accurate fault detection
  - It can be Adaptive (If the dynamics of the plant changes over time)

The fault detection algorithm can be integrated into the DoE algorithm in order to have an understanding of the normal behavior of the system

Limitations:

- In set membership approach there is the need of storing measurement data on the memory which could be problematic for embedded systems.
- Computation of the set  $\mathcal{S}$  that is required for the SMPC is difficult which makes it computationally expensive and hard to implement in fast dynamic systems. (a different cost function for the controller is also proposed which is easy to compute but it doesn't have the theoretical guarantees).



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Thank you for your attention



## Conclusion

- Publications
  - Under review
    - 2019 - Journal Automatica "Design of Experiments for Nonlinear System Identification: a Set Membership Approach"
  - Published
    - Karimshoushtari, M., Novara, C. and Spagnolo L., 2018 "Set membership fault detection for nonlinear dynamic systems" Book chapter IET.
    - Karimshoushtari, M. and Novara, C., 2019. A Data-Driven Model Predictive Control Approach to Lean NOx Trap Regeneration. *Journal of Dynamic Systems, Measurement, and Control*, 141(1), p.011016.
    - Karimshoushtari, M. and Novara, C., 2017, December. Lean NO x trap regeneration control: A data-driven MPC approach. In Decision and Control (CDC), 2017 IEEE 56th Annual Conference on (pp. 226-231). IEEE.
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    - Hemmatpour, M., Karimshoushtari, M., Ferrero, R., Montrucchio, B., Rebaudengo, M. and Novara, C., 2017, July. Polynomial classification model for real-time fall prediction system. In 2017 IEEE 41st Annual Computer Software and Applications Conference (COMPSAC) (pp.973-978). IEEE.
    - Novara, C. and Karimshoushtari, M., 2016, December. A data-driven model inversion approach to cancer immunotherapy control. In Decision and Control (CDC), 2016 IEEE 55th Conference on (pp. 5047-5052). IEEE.



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